Wzory trygonometryczne

Jedynka trygonometryczne

$$\sin^2 \alpha + \cos^2 \alpha = 1$$

Wzory na tangens i cotangens

$$\mathrm{tg}\alpha = \frac{\sin \alpha}{\cos \alpha}$$

$$\mathrm{ctg}\alpha = \frac{\cos \alpha}{\sin \alpha}$$

$$\mathrm{tg}\alpha \cdot \mathrm{ctg}\alpha = 1$$

Funkcje trygonometryczne podwojonego kąta

$$\sin 2\alpha = 2 \sin \alpha \cos \alpha = \frac{2\ \mathrm{tg}\alpha}{1 + \mathrm{tg}^2 \alpha}$$

$$\cos 2\alpha = \cos^2 \alpha - \sin^2 \alpha = 2 \cos^2 \alpha - 1$$

$$\mathrm{tg}2\alpha = \frac{2\ \mathrm{tg}\alpha}{1 - \mathrm{tg}^2 \alpha} = \frac{2}{\mathrm{ctg}\alpha - \mathrm{tg}\alpha}$$

$$\mathrm{ctg}2\alpha = \frac{\mathrm{ctg}^2 \alpha - 1}{2\ \mathrm{ctg}\alpha} = \frac{\mathrm{ctg}\alpha - \mathrm{tg}\alpha}{2}$$

Funkcje trygonometryczne potrojonego kąta

$$\sin 3\alpha = -4 \sin^3 \alpha + 3 \sin \alpha$$

$$\cos 3\alpha = 4 \cos^3 \alpha - 3 \cos \alpha$$

$$\mathrm{tg}3\alpha = \frac{3\ \mathrm{tg}\alpha - \mathrm{tg}^3 \alpha}{1 - 3\ \mathrm{tg}^2 \alpha}$$

$$\mathrm{ctg}3\alpha = \frac{\mathrm{ctg}^3 \alpha - 3\ \mathrm{ctg}\alpha}{3\ \mathrm{ctg}^2 \alpha - 1}$$

Funkcje trygonometryczne sumy i różnicy kątów

$$\sin(\alpha+\beta)=\sin\alpha\cos\beta+\sin\beta\cos\alpha$$

$$\sin(\alpha-\beta)=\sin\alpha\cos\beta-\sin\beta\cos\alpha$$

$$\cos(\alpha+\beta)=\cos\alpha\cos\beta-\sin\alpha\sin\beta$$

$$\cos(\alpha-\beta)=\cos\alpha\cos\beta+\sin\alpha\sin\beta$$

$$\mathrm{tg}(\alpha+\beta)=\frac{\mathrm{tg}\alpha+\mathrm{tg}\beta}{1-\mathrm{tg}\alpha\,\mathrm{tg}\beta}$$

$$\mathrm{tg}(\alpha-\beta)=\frac{\mathrm{tg}\alpha-\mathrm{tg}\beta}{1+\mathrm{tg}\alpha\,\mathrm{tg}\beta}$$

$$\mathrm{ctg}(\alpha+\beta)=\frac{\mathrm{ctg}\alpha\,\mathrm{ctg}\beta-1}{\mathrm{ctg}\beta+\mathrm{ctg}\alpha}$$

$$\mathrm{ctg}(\alpha-\beta)=\frac{\mathrm{ctg}\alpha\,\mathrm{ctg}\beta+1}{\mathrm{ctg}\beta-\mathrm{ctg}\alpha}$$

Wzory redukcyjne

$\sin(90^\circ+\alpha)=\cos\alpha$ $\quad$ $\sin(90^\circ-\alpha)=\cos\alpha$

$\cos(90^\circ+\alpha)=-\sin\alpha$ $\quad$ $\cos(90^\circ-\alpha)=\sin\alpha$

$\mathrm{tg}(90^\circ+\alpha)=-\mathrm{ctg}\alpha$ $\quad$ $\mathrm{tg}(90^\circ-\alpha)=\mathrm{ctg}\alpha$

$\mathrm{ctg}(90^\circ+\alpha)=-\mathrm{tg}\alpha$ $\quad$ $\mathrm{ctg}(90^\circ-\alpha)=\mathrm{tg}\alpha$

$\sin(180^\circ+\alpha)=-\sin\alpha$ $\quad$ $\sin(180^\circ-\alpha)=\sin\alpha$

$\cos(180^\circ+\alpha)=-\cos\alpha$ $\quad$ $\cos(180^\circ-\alpha)=-\cos\alpha$

$\mathrm{tg}(180^\circ+\alpha)=\mathrm{tg}\alpha$ $\quad$ $\mathrm{tg}(180^\circ-\alpha)=-\mathrm{tg}\alpha$

$\mathrm{ctg}(180^\circ+\alpha)=\mathrm{ctg}\alpha$ $\quad$ $\mathrm{ctg}(180^\circ-\alpha)=-\mathrm{ctg}\alpha$

$\sin(270^\circ+\alpha)=-\cos\alpha$ $\quad$ $\sin(270^\circ-\alpha)=-\cos\alpha$

$\cos(270^\circ+\alpha)=\sin\alpha$ $\quad$ $\cos(270^\circ-\alpha)=-\sin\alpha$

$\mathrm{tg}(270^\circ+\alpha)=-\mathrm{ctg}\alpha$ $\quad$ $\mathrm{tg}(270^\circ-\alpha)=\mathrm{ctg}\alpha$

$\mathrm{ctg}(270^\circ+\alpha)=-\mathrm{tg}\alpha$ $\quad$ $\mathrm{ctg}(270^\circ-\alpha)=\mathrm{tg}\alpha$

$\sin(360^\circ+\alpha)=\sin\alpha$ $\quad$ $\sin(360^\circ-\alpha)=-\sin\alpha$

$\cos(360^\circ+\alpha)=\cos\alpha$ $\quad$ $\cos(360^\circ-\alpha)=\cos\alpha$

$\mathrm{tg}(360^\circ+\alpha)=\mathrm{tg}\alpha$ $\quad$ $\mathrm{tg}(360^\circ-\alpha)=-\mathrm{tg}\alpha$

$\mathrm{ctg}(360^\circ+\alpha)=\mathrm{ctg}\alpha$ $\quad$ $\mathrm{ctg}(360^\circ-\alpha)=-\mathrm{ctg}\alpha$

Sumy i różnice funkcji trygonometrycznych

$$\sin\alpha + \sin\beta = 2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$$

$$\sin\alpha - \sin\beta = 2\cos\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2}$$

$$\cos\alpha + \cos\beta = 2\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$$

$$\cos\alpha - \cos\beta = -2\sin\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2}$$

$$\mathrm{tg}\alpha + \mathrm{tg}\beta = \frac{\sin(\alpha+\beta)}{\cos\alpha\cos\beta}$$

$$\mathrm{tg}\alpha - \mathrm{tg}\beta = \frac{\sin(\alpha-\beta)}{\cos\alpha\cos\beta}$$

$$\mathrm{ctg}\alpha + \mathrm{ctg}\beta = \frac{\sin(\beta+\alpha)}{\sin\alpha\sin\beta}$$

$$\mathrm{ctg}\alpha - \mathrm{ctg}\beta = \frac{\sin(\beta-\alpha)}{\sin\alpha\sin\beta}$$

$$\cos\alpha + \sin\alpha = \sqrt{2}\sin(45^\circ+\alpha) = \sqrt{2}\cos(45^\circ-\alpha)$$

$$\cos\alpha - \sin\alpha = \sqrt{2}\cos(45^\circ+\alpha) = \sqrt{2}\sin(45^\circ-\alpha)$$

Sumy i różnice jedności z funkcjami trygonometrycznymi

$$1+\sin\alpha = 2\sin^2\left(45^\circ+\frac{\alpha}{2}\right) = 2\cos^2\left(45^\circ-\frac{\alpha}{2}\right)$$

$$1-\sin\alpha = 2\sin^2\left(45^\circ-\frac{\alpha}{2}\right) = 2\cos^2\left(45^\circ+\frac{\alpha}{2}\right)$$

$$1+\cos\alpha = 2\cos^2\frac{\alpha}{2}$$

$$1-\cos\alpha = 2\sin^2\frac{\alpha}{2}$$

$$1+\mathrm{tg}^2\alpha = \frac{1}{\cos^2\alpha}$$

$$1+\mathrm{ctg}^2\alpha = \frac{1}{\sin^2\alpha}$$

Różnice kwadratów funkcji trygonometrycznych

$$\sin^2\alpha - \sin^2\beta = \cos^2\beta - \cos^2\alpha = \sin(\alpha+\beta)\sin(\alpha-\beta)$$

$$\cos^2\alpha - \sin^2\beta = \cos^2\beta - \sin^2\alpha = \cos(\alpha+\beta)\cos(\alpha-\beta)$$

Iloczyny funkcji trygonometrycznych

$$\sin\alpha\sin\beta = \frac{1}{2}[\cos(\alpha-\beta) - \cos(\alpha+\beta)]$$

$$\cos\alpha\cos\beta = \frac{1}{2}[\cos(\alpha-\beta) + \cos(\alpha+\beta)]$$

$$\sin\alpha\cos\beta = \frac{1}{2}[\sin(\alpha-\beta) + \sin(\alpha+\beta)]$$